

Peer Review Report

Manuscript Title: Functorial Topological Data Compression via Stratified Persistent Sheaves and Enriched Interleavings

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1. Summary and Scope of the Manuscript

This manuscript presents an ambitious and mathematically rigorous framework for topological data analysis (TDA) rooted in sheaf theory and stratification. The central construct is the stratified persistent sheaf (SPS), extending classical persistent homology to retain spatial attribution across filtrations. Innovations include a functorial treatment via MacPherson–Treumann equivalence, an enriched interleaving metric, a stability theorem, and a finite presentation theorem. A fully worked example and a Python implementation support the theoretical developments.

2. Strengths

- **Theoretical Depth and Originality:** Profound synthesis of homological algebra, sheaf theory, and topology, with original concepts like enriched interleaving distance.
- **Rigor and Precision:** Clear hypotheses, formal theorems, and carefully structured proofs.
- **Applicability:** The theory is relevant to multi-sensor fusion, stratified sampling, and hierarchical data.
- **Scholarly Apparatus:** Citations are well chosen, and inclusion of Python code boosts reproducibility.

3. Weaknesses and Recommendations

- **Length and Density:** Split into two papers to improve accessibility.
- **Entry Barriers for Non-Specialists:** Add intuitive overview and simplified examples.
- **Limited Experimental Evaluation:** Include real-data examples or case studies.
- **Undeveloped Universality Conjecture:** Move to future work or expand into formal categorical appendix.

4. Minor Corrections

Location	Issue	Suggested Correction
Throughout	“constructible sheaves with respect to \mathcal{C} ”	Clarify terminology in early sections
§IV.A	No diagram for evolution	Add stratum evolution diagram
§VII	Worked example text-heavy	Include labelled figures per step
Python Code	Minimal comments	Add comments clarifying filtration logic

5. Verdict and Recommendation

★ **Accept with Minor Revisions.** This manuscript presents a landmark development in sheaf-theoretic TDA. Restructuring and real-world validation would strengthen impact.

6. Annotated Summary for Editors

Criterion	Rating (1–5)
Originality of Concept	5
Mathematical Rigor	5
Clarity of Exposition	3.5
Relevance to Topological Data	5
Potential for Application	4
Suitability for Journal	5
Overall Recommendation	5 (Accept)